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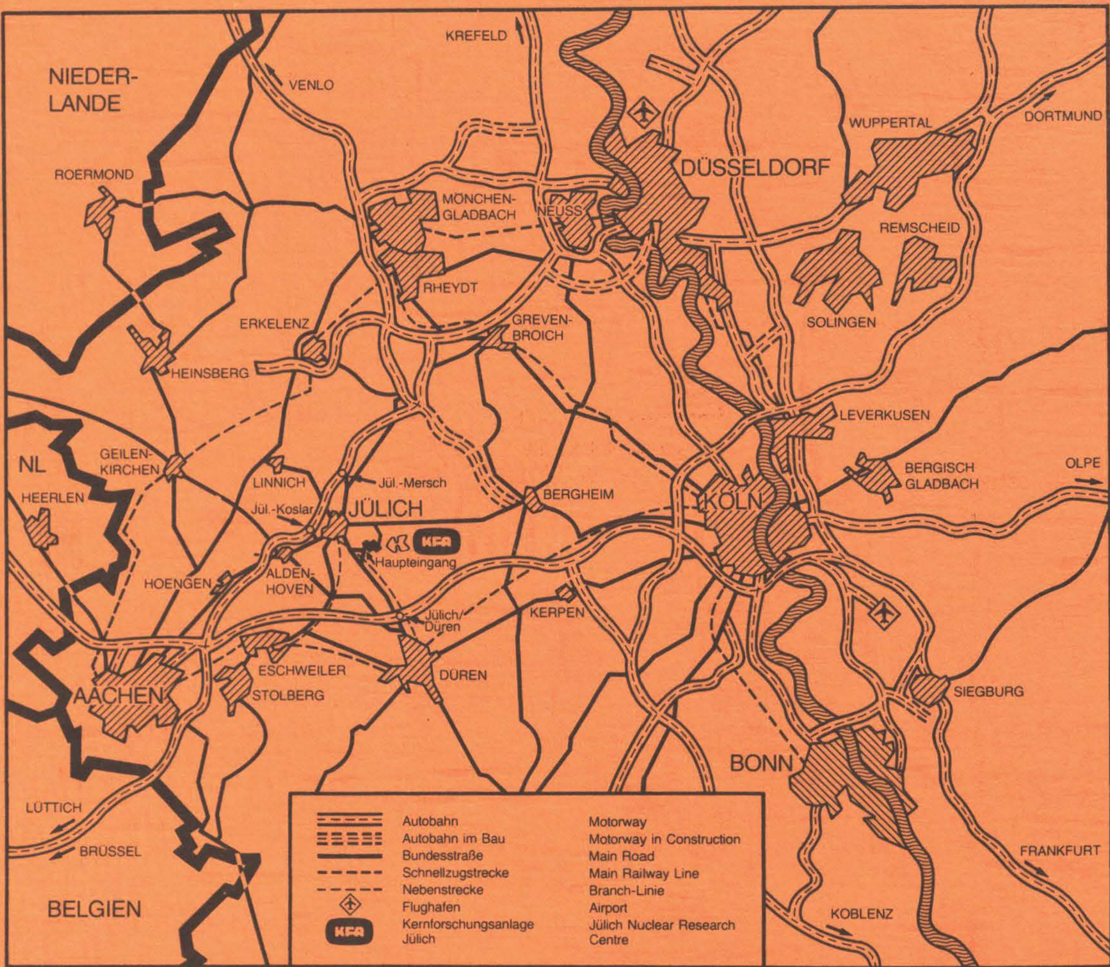

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Statistical Trend Analysis Methodology for Rare Failures in Changing Technical Systems

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ABSTRACT

A methodology for a statistical trend analysis (STA) in failure rates is presented. It applies primarily to relatively rare events in changing technologies or components. The formulation is more general and the assumptions are less restrictive than in a previously published version. Relations of the statistical analysis and probabilistic assessment (PRA) are discussed in terms of categorization of decisions for action following particular failure events. The significance of tentatively identified trends is explored. In addition to statistical tests for trend significance, a combination of STA and PRA results quantifying the trend complement is proposed. The STA approach is compared with other concepts for trend characterization.

INTRODUCTION

The drive of engineers to improve the functioning of a device or a larger technical system is as old as the engineering profession. Along with this drive to reduce the chances for failures goes the tendency to reduce and limit their consequences. Thus, with either failure-probability or consequences, or both being reduced during the development of technical systems, the "risk", i.e. the product of failure probability and associated consequences, is normally reduced even more. As the evolution of technologies is an ongoing process, one can normally expect a trend of declining failure rates and risk.

If the risk of a unit of a certain technology were a constant, invariable with time and the same for all units, the deployment of more and more units would imply a linear increase of the overall risk exposure. However, the continuing improvements (that are in part based on learning from operating experience including major failures) invalidate the premise. Thus, the long-term trend of failure rates (or risk) is not a linearly increasing function. It could even decrease if the (annual) reduction of total failure rate in the existing population exceeds the failure rate contribution of the new additions. The presentation of a methodology for the statistical evaluation of the longer-term trends, that can be deduced from the actuarial data on failures or accidents, is the main subject of this paper.

A first characterization of the operating experience, e.g. in terms of failures (within a certain category), can be obtained by forming an overall average rate for the entire operating period for a single system or component, for several systems or for the entire "population." If improvements are made in the systems, one is interested in their success that should subsequently be exhibited in the actuarial data as a trend of declining failure rates. Again, average values over the operating experience in temporal segments could be employed to give a first characterization of the trend. However, if the data are sparse, the formation of a sequence of average values may hardly be meaningful. Thus, a more sophisticated trend analysis procedure, based on the temporal spacings between individual events may be advised (statistical trend analysis, STA). A special version of such a methodology has been developed in Refs. 1 and 2. It is briefly reviewed in Sec. I.

Traditionally, the reduction of failure possibilities is achieved by applying general as well as system-specific experience through the various phases of the development of a technology, starting with small-scale experimental models, proceeding through prototype and demonstration units to commercial application on an increasing scale. Applications of good engineering practice and judgment in the general development and in correcting the causes of failures is combined with more mathematical reliability assessment techniques (see e.g. Ref. 3) and - especially for the nuclear reactor technology - with probabilistic risk assessment, PRA (see e.g. Refs. 4 through 7).

Detailed preanalysis with the PRA methodology leading to an "acceptable" design could be viewed as a superior alternative to the quasi-continuous system improvements through learning from operating experiences and failures. However, there are strong interrelations between the two approaches that may result in an "evolutionary pressure" toward continuing system improvement even in a case of an already "acceptable" design. These questions are addressed in Sec. II.

Section III then contains the generalized STA methodology followed in Section IV by a discussion of the evaluation of the statistical significance of a tentatively identified trend. In additon to a purely statistical evaluation of the spacing data, a combination of STA and PRA results for evaluating the complement of the trend is proposed to establish the trend significance by extensive application of prior information. Trend analysis, trend significance considerations and the comparative merits of different approaches to estimate current failure rates are illustrated for the U.S. nuclear power technology as an example. Finally, the essential aspects are summarized in a concluding section.

I. STATISTICAL EVALUATION OF MAJOR FAILURES

The first version of a statistical trend evaluation of failure events (STA) - along the line pursued here - was published in 1979 under the title "Statistical Evaluation of Major Human Errors During the Development of New Technological Systems" (Ref. 1). The aim was to provide an evaluation methodology for the learning process that accompanies normally the early evolution of new technologies. Often, some important aspects in the design of the hardware or the operational instructions are overlooked or misjudged during this phase. These deficiencies tend to soon become apparent, causing a disruption in the operation, a major failure or even an accident. Subsequently, the just revealed deficiencies are normally eliminated by proper design modifications such that a recurrence of the respective failures is precluded or at least considerably reduced in its likelihood.

In preparing the application of this methodology to the evolution of commercial nuclear power in the U.S. several additional aspects needed for a practical application have been introduced in Ref. 2. At first the failure events to be dealt with had to be characterized more precisely:

The normal design process includes the consideration of many failure modes leading to proper design adjustments such that their appearence as failure events is either precluded or sufficiently unlikely. What remains then is a residuum of failure modes that is to be distinguished from those that have been overlooked or misjudged. The terms IFM and UFM have been used in Ref. 2, designating a priori "identified failure modes" and "unidentified failure modes" respectively.

As with most classifications in two broad categories, there is a variety of cases in between. Actually, a continuous transition from clear IFM- into clear UFM-events is employed below to conceptually link probabilistic with event-oriented statistical analyses.

The basic model assumptions of Ref. 1 were that deficiencies (of the UFM-type) are built in a system from the beginning, and their chance of occurrence can be described by constant values λ_k. Furthermore, it was assumed that - after one of these failures had occurred - the just revealed failure mode would be designed out by proper system modifications, reducing its chance of recurring to zero. In addition the failure modes were assumed to be statistically independent.

Naturally, neither the value of any of these λ nor their total number can be known. However, since they are built in with a constant occurrence rate, they all have a chance to occur at any time. Thus, it is the sum of all (still) existing λ that determines the present total rate of occurrence, L. If the K-1 values of λ, that have revealed themselves first, have already been eliminated, the present total rate of occurrence is given by

$$L_K = \sum_{k=K}^{\cdot\cdot} \lambda_k, \qquad (1)$$

where the dots indicate the unknown upper limit of the sum (if the remaining λ_k become smaller and smaller, an upper limit is irrelevant). In a sense, L_K is the "residual" occurrence rate that comprises the total of all UFM possibilities, measured by their λ_k , that are still left in the system.

The key aspect of this statistical evaluation method is that the total of the λ_k determines the rate of occurrence and thus the temporal spacing (ΔT_k)*; therefore the spacing, ΔT_k, of actual events allows us to infer or estimate the current residual occurrence rate L_K by proper inversion of the ΔT_k.

The inverse of the properly <u>averaged</u> spacings gives the L_K-estimate. The inverse of <u>individual</u> spacings however reflects the statistical fluctuations

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* *The operating time of a single unit or of a multitude of units is denoted by "T", whereas lower case "t" is used for the time variable.*

that could be very large as shown in Section 2. The application of the individual Δt_k for estimating L_k could therefore suggest an increase, if a particular spacing is incidentally small. However, if there is prior information that suggests system improvements, here described by λ-elimination in Eq. (1), one may want to analyze the data for a "declining failure rate" (DFR). Thus, spurious increases need to be eliminated.

The isotonic regression procedure, introduced in 1972 by Barlow et al. (Ref. 8) appears to be ideally suited for the elimination of statistical perturbations from a tentatively assumed trend, since it applies the least modification of the original data, the ΔT_k values (see also Ref. 1). It approximates the ΔT_k by a sequence $\Delta\hat{T}_k$ in a least square sense, subjected to the order restriction of non-decreasing estimates. It merely eliminates the statistical decreases of the $\Delta\hat{T}_k$ (and thus the corresponding increases of the L_k-estimates), in order to identify the magnitude of a tentatively assumed trend (such as "declining failure rate", DFR). With $\Delta\hat{T}_k$ (k = 1...K) being the result of the isotonic regression, the L-estimates are obtained as

$$\hat{L}_k = (\Delta\hat{T}_k)^{-1} \qquad k = 1...K, \tag{2}$$

which describe a declining failure rate

$$\hat{L}_1 > \hat{L}_2 \geq \cdots \geq \hat{L}_K. \tag{3}$$

The equal sign that may appear between some of the $\hat{L}_k$-values reflects the fact that spurious increases are removed by the least possible adjustments.

If in a special case, the ΔT_k should increase monotoneously by themselves, no isotonic regression adjustment is needed. Then, the simple inversion of the ΔT_k-values yields a monotoneous L_k-estimate. In order to distinguish it from Eq. (2) it is called "non-isotonic" estimate, denoted by $\tilde{L}_k$:

$$\tilde{L}_k = (\Delta T_k)^{-1}. \tag{4}$$

If the ΔT_k are not monotoneously increasing, Eq. (4), i.e. the $\tilde{L}_k$-values, reflect the spurious increases.

The entire set of spacing data could also be combined to yield an average occurrence rate $\bar{L}$ with

$$\bar{L}_K^H = K\left(\sum_{k=1}^{K} \Delta T_k\right)^{-1} = \frac{K}{T_K} \quad , \tag{5}$$

where T_K is the total operation time. Forming $\bar{L}_k^H$ after each failure event gives a quantity that is called here the historical average rate, $\bar{L}_k^H$. It also reflects a trend, but normally in a considerably reduced way. A comparative discussion of $\bar{L}_k^H$ and $\hat{L}_k$ is included in Sec. V.

Apparently, the trend analysis with isotonic regression yields more detailed information from the same data than the average characterization, Eq. (5). This is possible since additional information has been used, i.e. the data are analyzed in conjunction with "prior information," namely that a decreasing trend can be expected because of the elimination of recognized failure possibilities or a reduction of their likelihood. The tentatively identified trend must then be subjected to a test of its statistical significance (see Section IV). No explicit allowance for possible learning is made in the historical average value $\bar{L}_k^H$. Every failure contributes "one" to the numerator (K) of Eq. (5) and remains there for all times, not accounting for design modifications, even if they preclude a recurrence.

II. STATISTICAL VS. PROBABILISTIC CONSIDERATIONS ON FAILURES

The differences and interrelations of event-oriented safety analysis (STA) and probabilistic risk assessment (PRA) reflect the methodological and conceptual differences of statistics and probabilistics:

- Probabilistic evaluations employ the asymptotic concepts "probability" and "average rate." Actuarial data are used to estimate or construct these asymptotic quantities. Limitations of an actual data base appear then in form of "inaccuracies" of the estimated average concepts, described by standard deviations or confidence intervals.
- Statistics on the other hand is the methodology of drawing inferences directly from the data base, without conceptually employing and without necessarily aiming at average concepts.

If a trend is superimposed on a sparse data base, e.g. because the system or the technology keeps changing while a sparse data base is assembled, average concepts may be of questional descriptive value. One may then obtain more revealing information by the evaluation of the trend as such.

In the extreme case that failures or accidents are so unlikely that no actuarial data exist at all, an evaluation must resort to the construction or synthesis techniques of the probabilistic approach.

It therefore appears that the domain for an application of statistical trend analysis techniques is between the <u>frequent</u> small failures: that provide an extensive data base for the formation of a sequence of average values, and the <u>extreme unlikely</u> failures for which no actuarial data exist at all. Thus statistical trend analysis (STA) appears to be called for in the evaluation of <u>sparse data for changing systems</u>, where the "system" may be a larger unit or merely a component of it or alternately a population of units such as an entire technology.

The discussion of the interrelations of STA and PRA is based on the consideration of the timing uncertainty of actuarial events. It is assumed that - through extensive preanalysis of the safety characteristics of a design - all "unacceptable" identified failure modes (IFM) have been eliminated. The remaining or "residual" risk is deemed "acceptable."

The term "acceptable" in this context does not reflect the risk perception aspects. It merely refers to a set of design decisions which distinguish between "accepted" and "unaccepted" failure modes. Still the term "acceptable" is used here (rather than the more precise term "accepted") to indicate the judgmental component in safety decisions.

The design decisions that result from these acceptability considerations form the technical basis for the quantification with PRA. Let $\bar{L}$ be the PRA-calculated acceptable average residual occurrence rate for failures within a certain category and $\bar{C}$ the corresponding average consequence. Their product gives then the corresponding average (residual) risk rate,

$$\bar{R} = \bar{L}\ \bar{C},$$

where the notation indicates the conceptual nature of these quantities as average or expected values.

The uncertainty of the PRA results is expressed in probability distributions around $\bar{L}$ and $\bar{C}$. Both are often assumed in form of log-normal distributions. The magnitude of the uncertainty is often expressed by specifying a confidence interval, containing e.g. 90 % of the distribution integral. The ratio, r_{90}, of the upper and lower limits is used here for the band width characterization:

$$r_{90} = \left[\frac{\text{upper limit}}{\text{lower limit}}\right] \text{ of 90 \% confidence band.} \tag{6}$$

Actuarial data pertaining to individual events are related here to the calculated expected $\bar{L}$ and $\bar{C}$ values:

Let us consider the first occurrence of a particular failure, with a consequence C_1, occurring at the operation time ΔT_1 (after a proper reference time, $T = 0$; i.e. $\Delta T_1 = T_1$). Then, the consequence C, and the non-isotonic estimate for L, i.e.

$$\tilde{L}_1 = \frac{1}{\Delta T_1} \tag{7}$$

will be compared with $\bar{C}$ and $\bar{L}$ in an L-C plane. The estimate $\tilde{L}_1$ will be called "apparent likelihood."

Figure 1 shows the L-C plane in a log-log representation with both variables related to their respective expected values $\bar{L}$ or $\bar{C}$, i.e. $l = \log(L/\bar{L})$ is plotted vs. $c = \log(C/\bar{C})$. The expected value is indicated as heavy circle at (0, 0). The risk equals the acceptable value, $\bar{R}$, along a straight line with a slope of minus one, indicated as a heavy dashed line; it projects the "acceptable" risk, $\bar{R}$, to higher and lower consequences with inversely varying occurrence rates.

An actual event, with the consequence C_1 and the apparent likelihood $\tilde{L}_1$, is represented by a point on the L-C plane. The corresponding "apparent risk," $\tilde{R}_1$,

$$\tilde{R}_1 = \tilde{L}_1 C_1 \tag{8}$$

may be above or below the accepted risk line $\bar{R}$.

As noted above, the calculation of the expected values $\bar{L}$ and $\bar{C}$ is not entirely accurate and the inaccuracies are represented as probability distribution functions (pdf) around $\bar{L}$ and $\bar{C}$.

However, the major uncertainty in a comparison of an actual event with expected values could just come from the statistical nature of failure events, from the fact that they may occur at any time. Given that the system has not failed yet, the probability that it will do so in the

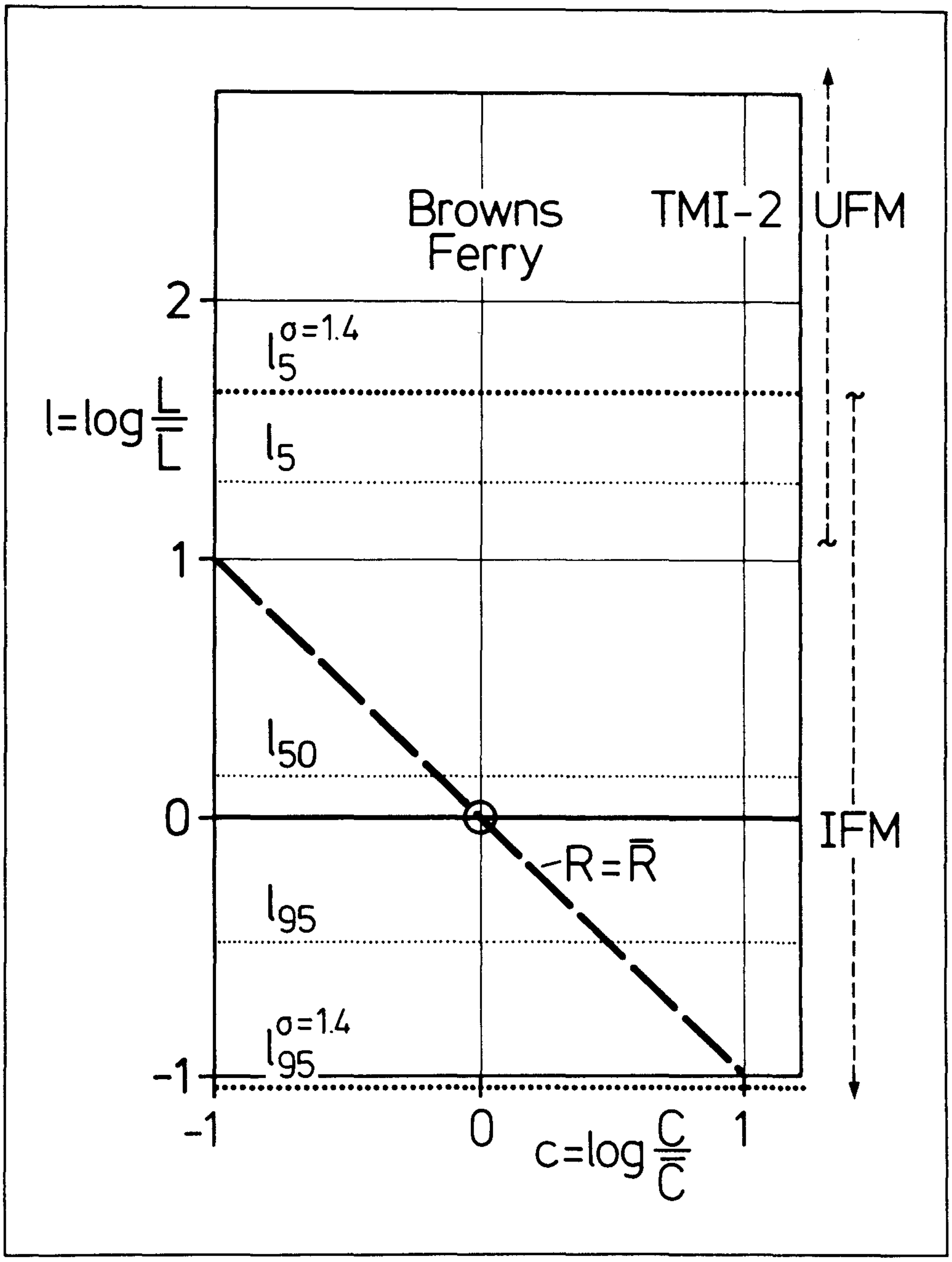

Fig. 1: Frequency vs. consequence diagram with confidence bands of the exponential and modified exponential density functions, $f(t)$ and $f_\sigma(t)$ respectively.

next interval dT is constant: $\bar{L}dT$, when $\bar{L}$ is assumed to be constant. The corresponding density function, f(T), is exponential:

$$f(T) = \bar{L}\, e^{-\bar{L}T}. \tag{9}$$

The "mean time to failure" for Eq. (9) is

$$\Delta\bar{T} = \frac{1}{\bar{L}}. \tag{10}$$

Also the spread in time values that results from the exponential distribution Eq. (9), without any uncertainty in $\bar{L}$ can be characterized by a 90 % confidence interval. The ratio of the corresponding upper to lower limits is denoted r^{ex}_{90}, to indicate that it results from the exponential distribution alone. The three time values for the median and the limits of the 90 % confidence interval for the exponential are given by

$$\left.\begin{aligned} T_5 &= \frac{1}{\bar{L}} \ln \frac{1}{.95} \approx \frac{0.05}{\bar{L}} \\ T_{50} &= \frac{1}{\bar{L}} \ln \frac{1}{.5} \approx \frac{0.69}{\bar{L}} \\ T_{95} &= \frac{1}{\bar{L}} \ln \frac{1}{.05} \approx \frac{3.0}{\bar{L}} \end{aligned}\right\} \quad r^{ex}_{90} \approx 60 \tag{11}$$

For plotting these T-limits and the apparent likelihood values per Eq. (7) in Fig. 1, all quantities are related to $\bar{L}$:

$$\tilde{l} = \log \frac{\tilde{L}}{\bar{L}}, \tag{12a}$$

and for the percentile limits ($T_{\%}$) of Eq. (11):

$$l_{\%} = \log \frac{(1/T_{\%})}{\bar{L}} = \log \frac{\Delta\bar{T}}{T_{\%}}, \tag{12b}$$

The corresponding l-values are shown in Fig. 1 as horizontal dotted lines, denoted by l_5, l_{50}, and l_{95} respectively.

As noted in Eqs.(11), the 90 % confidence interval of the exponential spreads over a factor of 60. However, this spread is not symmetrical about the mean; it extends a factor of 20 toward the low t side and only a factor of 3 toward larger t . This is reflected in the range in which the apparent failure rates, $\tilde{L}$, are to be expected. Their range then exceeds the mean by a factor of 20 on the upside and only a factor of 3 on the downside.

In addition to the spread of the exponential, one has to include the uncertainty distribution of L around $\bar{L}$, for which a log-normal distribution is assumed $\left[p_{ln}(\frac{L}{\bar{L}};\ \sigma)\right]$ with a standard deviation σ. Averaging the exponential with p_{ln} results in a "modified exponential" density function, $f_\sigma(T)$:

$$f_\sigma(T) = \int_{-\infty}^{\infty} L\ e^{-LT}\ \ p_{ln}(\frac{L}{\bar{L}};\ \sigma)\ d\ln\frac{L}{\bar{L}}. \tag{13}$$

For calculational details see Appendix 1. It is seen that $f_\sigma(T)$ depends on $\bar{L}$ as parameter in the same way as f(T) on L; it has $\bar{L}$ as factor and depends on $\bar{L}T$. In the terminology of Eq. (13), f(T) per Eq. (9) is equal to $f_0(T)$:

$$\lim_{\sigma \to 0} f_\sigma(T) = f_0(T) = f(T)$$

The effect of averaging an exponential density function over an uncertainty distribution for L becomes qualitatively evident by considering L-values that are larger or smaller than $\bar{L}$. The larger L-values lead to steeper the smaller L-values to slower decreasing exponentials than given by Eq. (9). Figure 2 shows a comparison of $f(T) = f_0(T)$ with three modified exponentials, $f_\sigma(T)$, as defined by Eq. (13). The three log-normal uncertainty distributions have 90 % confidence interval band widths with upper to lower

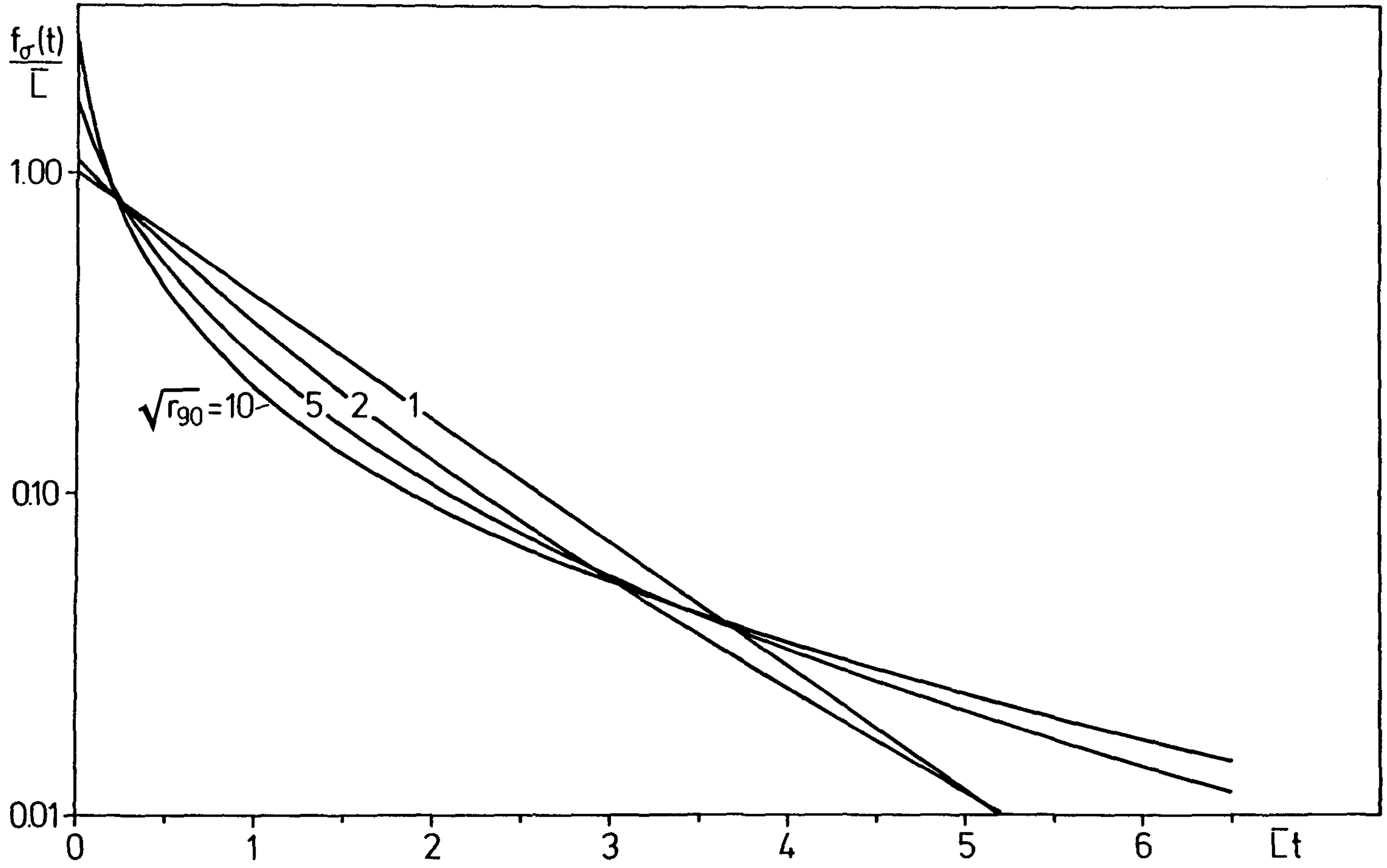

Fig. 2: Modified exponential density functions, $f_\sigma(t)$, for $\sigma = 0.421$; $\sigma = 0.978$; $\sigma = 1.400$ compared with $f(t) = f_0(t)$. Plotted is $f_\sigma/\bar{L}$ as function of $\bar{L}t$. The three σ-values correspond to ratios of upper to lower limit of the 90 % confidence intervals of the log-normal distribution for L being $r_{90} = 4$, 25, and 100 respectively; $r_{90} = 1$ for $\sigma = 0$, given $f_0(t)$.

limit ratios of r_{90} = 4, 25, and 100 respectively. The corresponding σ-values are σ = 0.421, σ = 0.978 and σ = 1.400 respectively.

The derivations of f_σ and f_0 are as qualitatively expected. The values at T = 0 are increased; $f_\sigma(0)$ is given by Eq. (58) of App. 1. The three values are 1.09, 1.61 and 2.65 times $f_0(0) = \bar{L}$. The middle part of the modified density function is considerably lowered as compared to the unmodified one, whereas at large times, $f_\sigma(T)$ decreases much slower than f(T) which shows the effect of L-values that are much smaller than $\bar{L}$.

The modification of the exponential distribution by a variation in L will affect the width of the confidence band. Apparently, the 5 % integral at the left end of the highly skewed distributions will be reached at smaller times, where the decrease is roughly proportional to the increase of $f_\sigma(0)$ as compared to $f_0(0) = \bar{L}$. Figure 3 depicts as functions of σ the variations of the limits of the 90 % and 68 % confidence intervals as well as the median values (50 %). The 90 % confidence band limitations for r_{90} = 100 are also reflected in the corresponding $L/\bar{L}$-band of Fig. 1. The T-value at the 5 % integral leads to a large apparent L, about 50 times $\bar{L}$; the 95 % integral appears as a low L-value at about 0.1 $\bar{L}$. Thus, the 90 % confidence band for the modified exponential (with r_{90} = 100) extends over a factor of 500. This is only about 8 times more than for the exponential distribution alone.

The lower part of Fig. 1 contains events that are not too far away from expected and thus identified failure modes. Events for which the apparent likelihood is much larger than the expected one, i.e. failures that are quite unexpected would be in the upper part of Fig. 1. If they were nearly totally unexpected ($\bar{L} \lll L$) they would be at even larger l-values than shown in Fig. 1. However, most UFM-events are not totally unexpected; the element of surprise is more in the apparent likelihood, rather than in the type of event per se. Approximate IFM and UFM domains,

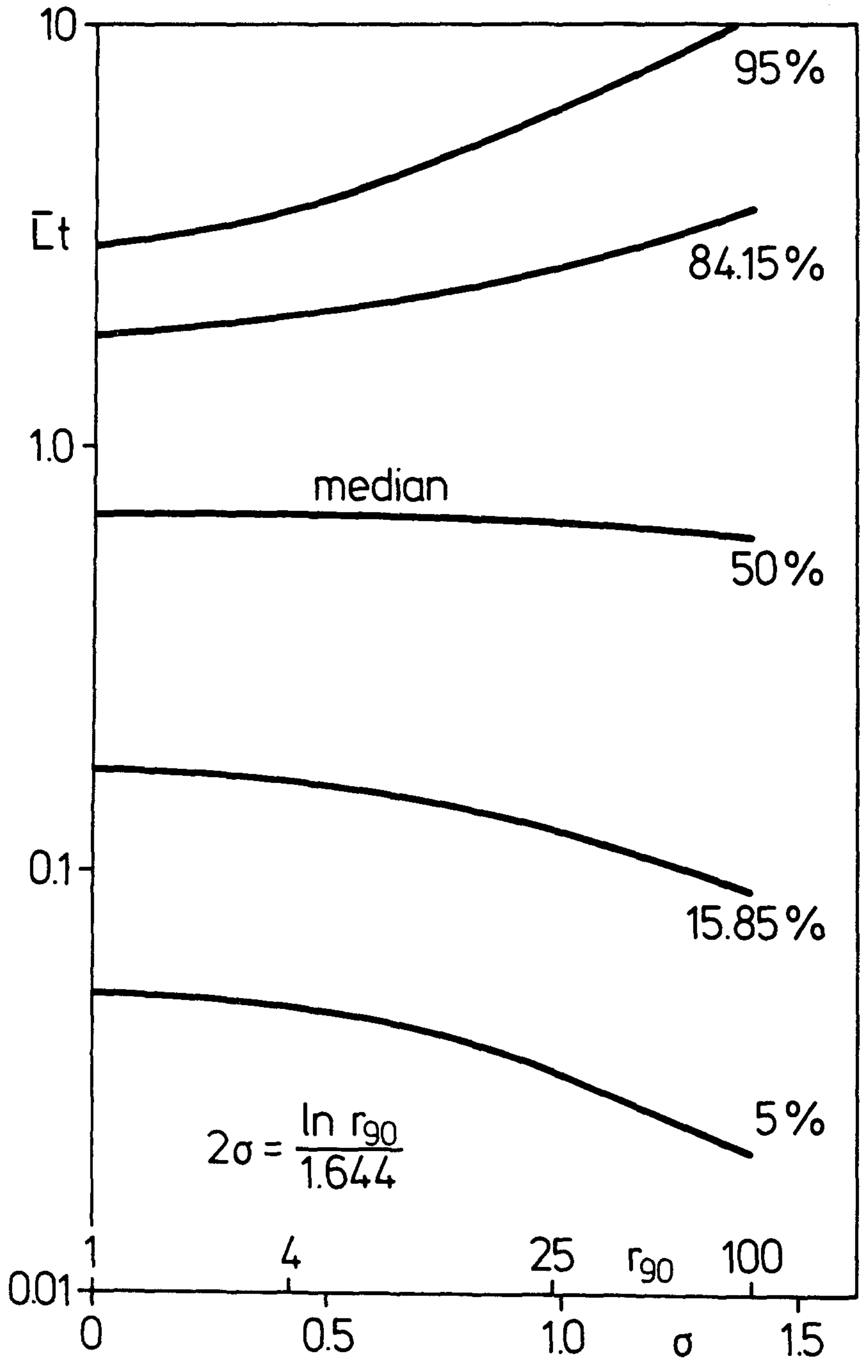

Fig. 3: Modified exponential density function, $f_\sigma(t)$: Median values, limits of the 68.3 % and 90 % confidence intervals as function of σ. The relative width r_{90} of the 90 % confidence band are indicated on the σ-axis.

overlapping to some extent, are indicated on the right of Fig. 1, viewed to be connected through a gradual transition rather than being disjunct domains.

A case in point for the not totally unexpected nature of UFM events are the two failures on U.S. nuclear power reactors, the Browns Ferry fire and the TMI-2 accident (see Sec. 5). A cable fire is not a totally unexpected incident nor is a "small LOCA" (loss of coolant accident), where "small" refers to the size of the opening through which coolant escapes. Nevertheless, the apparent likelihood, the magnitude of the (financial) consequences and the appearance of a "design" problem in either hardware or operational instructions qualify both incidents as UFM-events (comp. Ref. 2). The qualitative location of both accidents on the L-C plane is indicated in Fig. 1.

A major aspect in the consideration of the relation of the probabilistic and event-oriented approaches is that after each failure a decision must be made on the course of action. In Ref. 2, the course of action has been made the basis for an accident classification, that also led to the distinction between IMF and UFM types of events.

On the low end of the failure spectrum one will likely decide on "repair action only", (category IV of Ref. 2). By implementing this decision, one restores the likelihood of the failure to the value it had before and establishes a basis for forming an average failure rate.

The next category of failures (category III) is characterized by the decision that "more-than-repair action" is called for. Whatever this action may consist of - whether hardware or software alterations are carried out only in the failed unit or are spread to other units of this technology - the likelihood of a recurrence is reduced.

The events in category II of Ref. 2 are even more serious and reveal through their apparent likelihood or their actual consequences a flaw in the previously accepted design. Then design modifications are called for such that a recurrence is either designed out or made at least much less likely.

If in addition to the realization that a design modification is called for, the accident led to fatalities among the public, it was considered to be part of the category I.

This categorization of failures or accident according to the subsequent course of action can be used to subdivide the L-C plane. The apparent likelihood and the actual consequences represent the new information that has been provided by the actual failure event.

The domain for "repair action only" can be limited by the acceptable risk line, $\bar{R}$, as indicated in Fig. 4.

If, however, the apparent risk, $\tilde{R}$, was larger or even considerably larger than $\bar{R}$, one will have to determine whether a design flaw (UFM event) was more or less obviously involved in the accident. In this case a design modification would be called for.

But if the failure for which $\tilde{R} >> \bar{R}$ is clearly an IFM event, the decision is more difficult. Considering the large possible spread in the apparent likelihood, the specific event could well have an acceptable $\bar{L}$ that might prevail asymptotically. Arguing on the basis of probabilistic considerations alone, one could decide to wait for more failures to occur so that average values $\bar{L}$ and $\bar{C}$ can be better established. But it appears more likely that a R-value that is considerably larger than $\bar{R}$ will instigate more-than-repair action or even the design modification course of action. Approximate domains for these courses of action are indicated in Fig. 4. The magnitude of these domains could be different for different technologies. The categorization in Fig. 4 is based upon Ref. 2 which addressed the nuclear power technology.

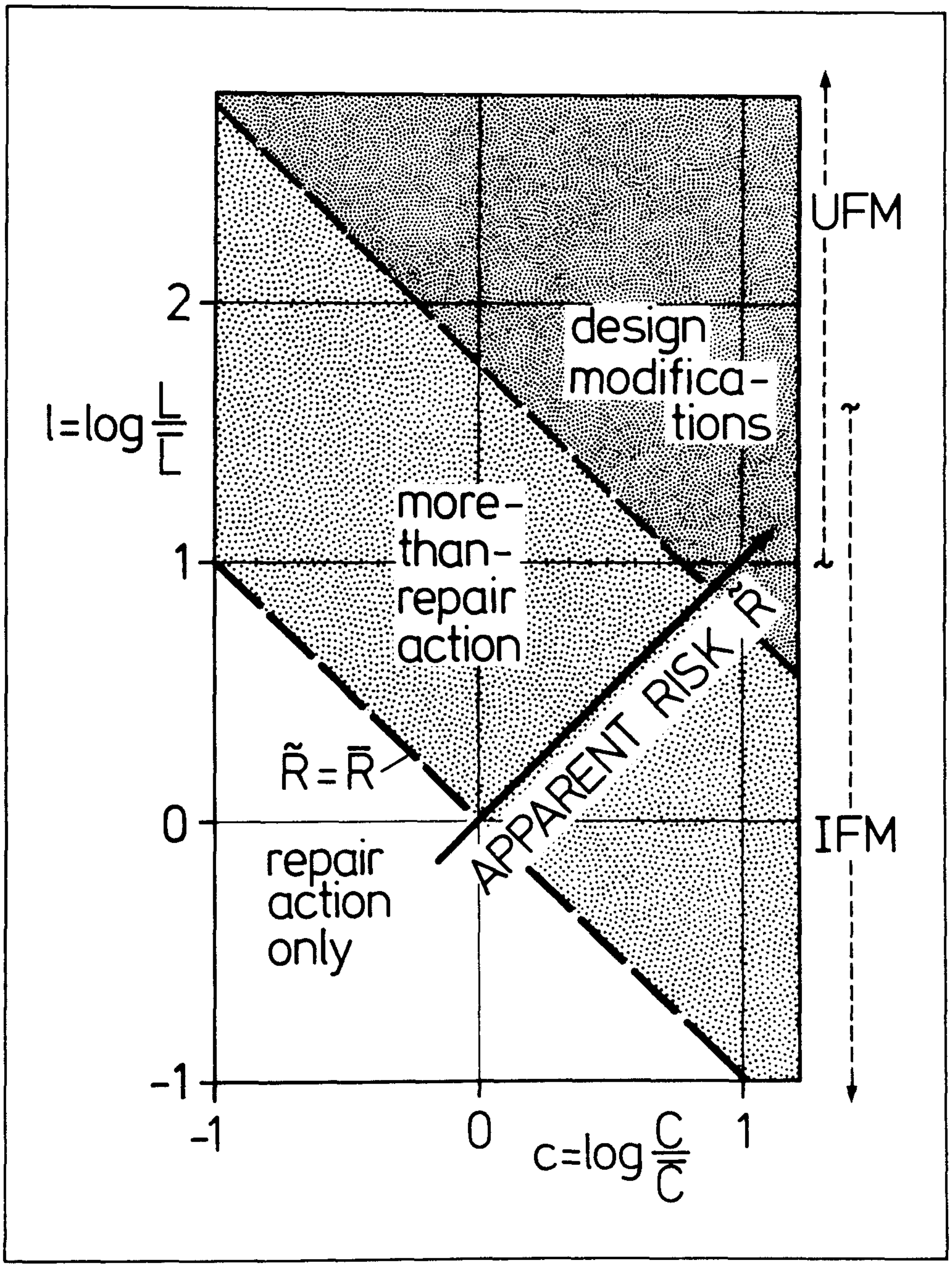

Fig. 4: Frequency vs. consequence diagram with lines of constant apparent risk, subdividing the three action domains "repair only", "more-than-repair" and "design modifications".

The main conclusions to be drawn from this discussion of courses of action and their representation on an L-C plane are the following:

- Due to the large spread of the apparent likelihood and risk ($\tilde{L}$ and $\tilde{R}$), especially on the upside, a large fraction of IFM events can be expected to occur with an $\tilde{R}$-value considerably above the acceptable value $\bar{R}$.
- In many of these cases the decision maker can be expected to call for more-than-repair action.
- Thus, in addition to system improvements following UFM accidents, one can expect more-than-repair action also for a considerable number of IFM-events.
- In effect then, the large variability of the outcomes of failure events leads to decisions that exert an evolutionary pressure toward system improvement.
- In any case, when systems are modified based on the experience for single or few failure events, the basis for establishing average rates, $\bar{L}$, or consequences ($\bar{C}$) by actuarial data erodes. Then, a statistical, event-oriented analysis aiming at a trending investigation should be employed for the evaluation of the technical progress as exhibited by the actuarial data on failures.
- Under the influence of an ongoing evolutionary pressure toward system improvement, risk seems to become a dynamic concept rather than being a static one. In risk dynamics investigations one would then explicitly address the parameters and forces that affect the dynamics behavior, especially the sign and the rate of changes of risk and failure rate, quantities that could be more important than some rather uncertain absolute values.

III. GENERALIZED MODEL FOR STATISTICAL TREND ANALYSIS USING ISOTONIC REGRESSION

III.1 Failure Rates and Risk

The earlier version of the STA model is reviewed in Sec. I. It pertained to major UFM events that were considered so serious that recurrence had to be precluded by proper system modification. The generalized model of an event-oriented failure analysis to be presented here can also be applied to IFM incidents. It needs no longer to be assumed that subsequent system modifications have to eliminate the corresponding λ. The trend search options have also been generalized. Decreasing as well as increasing failure rate trends can be searched for and if needed, possible indications of a trend reversal can be investigated.

The primary applications of this procedure is to the analysis of failure data of systems that are on the one hand

- sufficiently complex so that system modifications are made on them, on the other that
- failures are infrequent, so that the assemblage of a sufficient data base for the calculation of average occurrence rates is not meaningful between changes in the system's safety characteristic.

The basic model assumptions are that the potential failures can be described by rates, λ_{ik}, where the index i refers to the individual unit and k to the type of failure, and that the appearance of any one of these failures is a statistical phenomenon.

Furthermore, the λ_{ik} are assumed to be independent of time. Any variation of the occurrence rates, λ_{ik}, be it a reduction through retrofitting or an increase due to age-effects is formally described by application of alteration factors: $a_{ik}^{\lambda}(T)$; see Eq. (19) .

Occasionally, a change in a system leads to a new failure possibility; the corresponding a-value would then be zero prior to that time and unity thereafter. The new failure possibility may have been built in - along with a repair - inadvertently, or a change in a system might now allow a failure to occur which could not happen before. This is a typical violation of the statistical independence of failures. A practical example is the "failure repair" (debugging) in computer software that makes program parts accessable (including the errors in these parts) that could not be reached before the "repair " (see Ref. 9).

It is noted here that the isotonic regression estimation as such does not make use of the statistical independence. Only in the test of the trend significance is the assumption of statistical independence employed to determine, e.g. the probability for a zero trend hypothesis to be descriptive of the actuarial data.

Since the event-oriented analysis yields results only at discrete points along the time axis, at the times of failures, only average values over the time intervals between failures are needed. Let $a_{ik}^{\lambda K}$ be the average over the K-th failure interval (ΔT_K):

$$a_{ik}^{\lambda K} = \frac{1}{\Delta T_K} \int_{T_{K-1}}^{T_K} a_{ik}^{\lambda}(T)\ dT. \tag{14}$$

If the $a_{ik}^{\lambda}(T) = 1$, the failure rate for the i-th system is given by:

$$\Lambda^{(i)} = \sum_{k}^{\cdots} \lambda_{ik}, \tag{15}$$

where the dots indicate an unknown upper limit in the number of possible failures.

For the description of the various units in a population of systems, one needs the individual start-up dates and operational schedules. Let the fraction of the K-th interval that the unit i operates be denoted by ω_{iK}. Then, $\omega_{iK} = 1$ if the unit i operates throughout the K-th interval. The total Λ_K is then given by

$$\Lambda_K = \sum_i \omega_{iK} \sum_k^{\cdots} \lambda_{ik} = \sum_i \omega_{iK} \Lambda^{(i)} \frac{\text{failures}}{\text{year}} . \tag{16}$$

Since the analysis is retrospective for each interval the ω_{iK} are known for the evaluation of the K-th failure.

The sum of the individual fractions of the operation of these systems in the K-th interval is

$$\Omega_K = \sum_i \omega_{iK} = \sum_i \frac{\Delta T_{iK}}{\Delta t_K} \frac{\text{plant years}}{\text{year}} , \tag{17}$$

where ΔT_{iK} is the operation time of the i-th unit in the K-th interval and Δt_K the calendar time between the K-th and the K-1st failures. If all ΔT_{iK} equal Δt_K, Ω_K is equal to the number of units considered. Dividing Λ_K by Ω_K gives the failure rate per plant year in the interval prior the K-th event

$$L_K = \frac{\Lambda_K}{\Omega_K} \frac{\text{failures}}{\text{plant year}} . \tag{18}$$

With alteration factors included which largely describe improvements derived from lessons from previous failures one has the respective reduced rates in the K-th interval:

$$\lambda_{ik}^K = a_{ik}^{\lambda K} \lambda_{ik}, \tag{19}$$

$$\Lambda_K^{(i)} = \sum_k^{\cdots} \lambda_{ik}^K , \tag{20}$$

$$\Lambda_K = \sum_i \omega_{iK} \sum_k^{\cdots} \lambda_{ik}^K = \sum_i \omega_{iK} \Lambda_K^{(i)}, \tag{21}$$

and L_K again from Eq. (18).

The isotonic regression procedure, briefly described in Sec. I, aims at estimating the rates L_K. The resulting estimate is denoted by $\hat{L}_K$. This STA estimate is to be interpreted as the <u>average failure rate during the recent time interval</u>, which is either ΔT_K alone, or an extended interval as far back as the isotonic regression includes ΔT_k-values in the averaging process. What the recent failure then provides is - in a sense - a "measurement" of the average rate in the preceding interval, though subject to statistical variations.

By applying the distinction between identified and unidentified failure modes (IFM and UFM respectively), one can subdivide all λ into these two groups:

$$\Lambda_K = \Lambda_K^{IFM} + \Lambda_K^{UFM} \tag{22}$$

Only the λ-values in the IFM part can be calculated by the synthesis procedure of PRA.

Since the consequences of accidents are treated here merely through a categorization of their severity, the evaluated failures rates, $\hat{L}_K$, apply only to frequencies of events in such a category. Nevertheless, the formal definition of the risk rate is given here also to establish the relation to the frequency formulas.

In order to obtain the risk, the failure rates need to be combined with estimates of the consequences C_{ik} for each path. The C_{ik} refer to any type of risk, such as repair cost or health hazard. However, in the same way as for the λ , there are also alteration factors a_{ik}^{cK} for the consequences that result from changes in the system through retrofitting.

As for the λ , these changes may in some cases also lead to an increase; the corresponding a^c would then be larger than unity.

For an individual technical system (a "unit") as well as a group or a "population" of such systems one obtains in analogy to the failure rates the following risk rates:

$$P_K^{(i)} = \sum_k \lambda_{ik}^K \, C_{ik}^K, \tag{23}$$

$$R_K = \frac{1}{\Omega_K} \sum_i \omega_{iK} \, P_K^{(i)} = \frac{P_K}{\Omega_K} \tag{24}$$

with $C_{ik}^K = a_{ik}^{cK} C_{ik}$ (25)

and $P_K = \sum_i \omega_{iK} \, P_K^{(i)}$. (26)

Here, R_K is the "residual risk", i.e. the risk remaining after the K-th failure, where the lessons learned from earlier failures have probably led to a reduction of the risk.

III.2 Isotonic Regression Analysis for Monotoneous Trends and Trend Reversals

As indicated in Sec. I, the isotonic regression search for a declining failure rate is performed as a search for increasing spacings ΔT_k. Mathematically one solves the minimization problem

$$\sum_{k=1}^{K} (\Delta\hat{T}_k - \Delta T_k)^2 = \text{minimum}, \tag{27}$$

subject to the order restriction

$$\Delta\hat{T}_1 \leq \ldots \leq \hat{T}_k \ldots \leq \Delta\hat{T}_K \; . \tag{28}$$

Alternately, in a search for increasing failure rates, the order restriction is just reversed:

$$\hat{\Delta T}_1 \geq \ldots \geq \hat{\Delta T}_k \ldots \geq \hat{\Delta T}_K. \tag{29}$$

The inverse $\hat{\Delta T}_k$ give the respective failure rates, per Eq. (2).

The searches for decreasing and increasing trends may be combined in a search for a <u>trend reversal</u>. The mathematical formulation is an extension of the minimum problem, Eq. (27), but both order restrictions appear in combination; the interval in which trend reversal begins is denoted by $\hat{\Delta T}_\kappa^\kappa$.

At first, all intervals are used individually in K trial solutions to determine the value $(S^2_{\kappa'})$ of the residual of the minimum problem

$$\frac{1}{K} \sum_{k=1}^{K} (\Delta T_k^{\kappa'} - \Delta T_k)^2 = S^2_{\kappa'} = \text{minimum} \quad , \tag{30}$$

subject to the order restriction

$$\Delta T_1^{\kappa'} \leq \ldots \leq \Delta T_{\kappa'}^{\kappa'} \geq \Delta T_{\kappa'+1}^{\kappa'} \ldots \geq \Delta T_K^{\kappa'} \quad , \tag{31}$$

(or reversed) for $\kappa' = 1, \ldots K$. Subsequently, the ΔT_κ^κ belonging to the smallest S^2_κ is designated as the trend reversal interval. The corresponding $\hat{\Delta T}_k^\kappa$-values are the final result of the search (indicated by "^"):

$$\hat{\Delta T}_1^\kappa \leq \ldots \leq \hat{\Delta T}_\kappa^\kappa \geq \hat{\Delta T}_{\kappa+1}^\kappa \geq \ldots \geq \hat{\Delta T}_K^\kappa \quad , \tag{32}$$

with κ from

$$\min (S^2_{\kappa'}) = S^2_\kappa \quad . \tag{33}$$

IV. SIGNIFICANCE OF TRENDS

The significance of a trend that has been tentatively identified by statistical trend analysis of the actuarial failure data can be evaluated basically in two different ways, by purely statistical means, or by directly employing the technical information on system changes.

IV.1 Statistical Trend Significance Evaluation

The statistical procedures consist of testing the data for a constant average spacing (null hypothesis, H_o) or of testing the significance of coefficients of a fit. The most common tests are concerned with linear fits, where the fit may be on any scale (e.g. linear or logarithmic).

The isotonic regression does not yield coefficients. Thus, it appears to be more appropriate to subject the data to the H_o-test mentioned above (see Refs. 8 and 1).

The result of the H_o test is expressed in terms of a percentile α, which - in a sense - is the probability for the set of data to be part of a sequence with a constant failure rate. If α is sufficiently small, H_o can be rejected and the alternate hypothesis (DFR) accepted. The evaluation of α is based upon the assumption of statistical independence.

In the case of a trend reversal analysis, the two branches have to be subjected individually to H_o tests; the declining and the increasing branches must be significant individually to suggest a real trend reversal. Let α^+ and α^- be the two percentiles. An α-value for the trend reversal case (say $\alpha^{\pm}$) can be obtained by combining α^+ and α^-. Naturally, the data requirements for establishing a trend reversal are more extensive than for a single trend.

Although the isotonic regression does not yield coefficients as a fit, a trend reversal evaluation can also be based on a confidence interval band around the estimate $\hat{L}_k$. Procedures to devise such a confidence band are under consideration. Presented here are only some fundamentals that can be readily derived from the exponentiality of the density function for individual events and from some special considerations for groupings that appear with the same $\hat{L}$-value in the isotonic regression.

The basic assumption is that the $\hat{L}_k$-values are the respective mean values of exponential density functions. This preliminary evaluation is, in addition, based on the assumption of statistical independence. Furthermore, the modification of the exponential density function that was introduced above for calculated $\bar{L}$-values, does not apply here.

A 68.3 % confidence band is deemed suitable for the trend significance evaluation. It will be shown below that for large groupings of intervals the 68.3 % interval approaches asymptotically the 2σ-interval of a normal distribution.

The time-boundaries for the 90 % confidence interval were given in Eq. (12). The corresponding L-values for the 68 % (short for 68.3) interval are

$$(L_k)_{16} = \frac{L_k}{0.174} \text{ and } (L_k)_{84} = \frac{L_k}{1.83} \tag{34}$$

where L_k is to be taken as the isotonic regression estimate $\hat{L}_k$.

If all $\hat{L}_k$ come out to be different, the 68 % interval can be obtained by applying Eq. (34) directly and independently to each L-estimate:

$$(L_k)_{16} \geq 68.3\ \%\ \text{interval of}\ L_k \geq (L_k)_{84}\ . \tag{35}$$

An example of this application is given in Sec. V.

For all-different $\hat{L}_k$, Eq. (35) yields a monotoneous confidence band, although the individual intervals for neighbouring events normally overlap. Then, an independent variation of L-values within this band could violate the trend. However, by imposing the order restriction, the isotonic regression affects the independency of the estimators by disallowing an L-estimate to be larger than the previous one (in an analysis for DFR). This requires an evaluation of the confidence band by properly correlating neighbouring interval. In general, one can expect the confidence interval to become narrower if this correlation is introduced.

If the $\hat{L}_k$ are not all different, the ΔT_k-values appear averaged for each group of equal $\hat{L}_k$. Let $\hat{L}^{(n)}$ be that common value:

$$\hat{L}^{(n)} = \hat{L}_k = \hat{L}_{k+1} \dots = \hat{L}_{k+n-1} \; . \tag{36}$$

The value $\hat{L}^{(n)}$ is then a particular finding in a historical sequence of events that cannot be repeated.

The question for the uncertainty distribution around $\hat{L}^{(n)}$ is inverted as compared with the question that leads to the Poisson distribution

$$P(n;\bar{n}) = \frac{(\bar{n})^n e^{-\bar{n}}}{n!} \; . \tag{37}$$

The latter gives the probability for a variety of particular values n, when $\bar{n}$ is the corresponding average. Here, n is a specific historical realization and we ask for the variety of $\bar{n}$ that could be associated with this particular n-value.

The case n = 1 has been discussed above. In terms of the general formulation used in the following, the corresponding exponential distribution is written as

$$g_1(\tau) = e^{-\tau} , \tag{37}$$

with

$$\tau = \hat{L}^{(1)} \Delta T_1 = \bar{n}. \tag{38}$$

The superscript indicates n = 1.

The average $\bar{n}$ (called here τ for short) is the average number of $\Delta T^{(1)}$-intervals that can be associated with the actual finding of n = 1, where $\Delta T^{(1)} = 1/\hat{L}^{(1)}$. Equation (37) gives the corresponding density function.

If n values of $\hat{L}$ agree, as in Eq. (36), one expects a smaller confidence band than for a single event. For n = 2 one obtains

$$g_2(\tau) = \tau\ e^{-\tau}. \tag{39}$$

Thus, the density function for two events is already less skewed than the exponential, Eq. (37). It approaches zero for decreasing value of τ (or $\bar{n}$).

In general, the density function for τ is the gamma distribution:

$$g_n(\tau) = \frac{\tau^{n-1}}{(n-1)!}\ e^{-\tau}, \tag{40}$$

with

$$\tau = \hat{L}^{(n)} \Delta T_n \tag{41a}$$

and

$$\Delta T_n = \sum_{k'=k}^{k+n-1} \Delta T_{k'}, \tag{41b}$$

The expectation value of $g_n(\tau)$ is

$$\bar{\tau} = \int_0^\infty \tau g_n(\tau)\ d\tau = n, \tag{42}$$

since the found n events have been associated with the expectation value.

For large n, the gamma distribution approaches the normal distribution around the maximum value τ_m:

$$g_{as}(\delta) = \frac{1}{\sqrt{2\pi\ \sigma^2}}\ e^{-\frac{\delta^2}{2\ \sigma^2}}, \tag{43}$$

with

$$\delta = \frac{\tau - \tau_m}{\tau_m}, \tag{44a}$$

$$\tau_m = n-1 \tag{44b}$$

and

$$\sigma = \frac{1}{\sqrt{\tau_m}} \approx \frac{1}{\sqrt{n}} \; . \tag{44c}$$

The median and the 68.3 % interval for the gamma distribution are depicted in Fig. 5 for n = 1 through 8, all quantities related to the mean value $\bar{\tau} = n$. The median slowly approaches the mean as the distribution becomes more symmetrical and approaches the Gaussian, Eq. (43). The 68.3 % intervals are quite symmetric on the linear scale of Fig. 5, already for n = 1.

In general, if the early and later confidence intervals do not overlap and are well separated, one will consider a trend to be statistically significant. Procedures for a quantification of the significance for confidence intervals around an isotonic regression result are under consideration.

IV.2 System Improvement Assessment and Trend Complement

If through repair action after a failure the previous state of the system, including the chance for a recurrence of that particular failure, has been fully restored, the failure rate should remain constant (assuming that system deterioration is prevented by proper maintenance). Thus, a declining failure rate can only result from the "more-than-repair" course of action after failures, or from improvements that are unrelated to the failure in existing systems. Thirdly, a decrease in the failure rate per operating year can be achieved by deploying new safer systems and by decommissioning obsolete ones. The improved safety of new units may in part result from lessons learned from earlier failures. All three effects are discussed in the following and are combined to provide a computed estimate

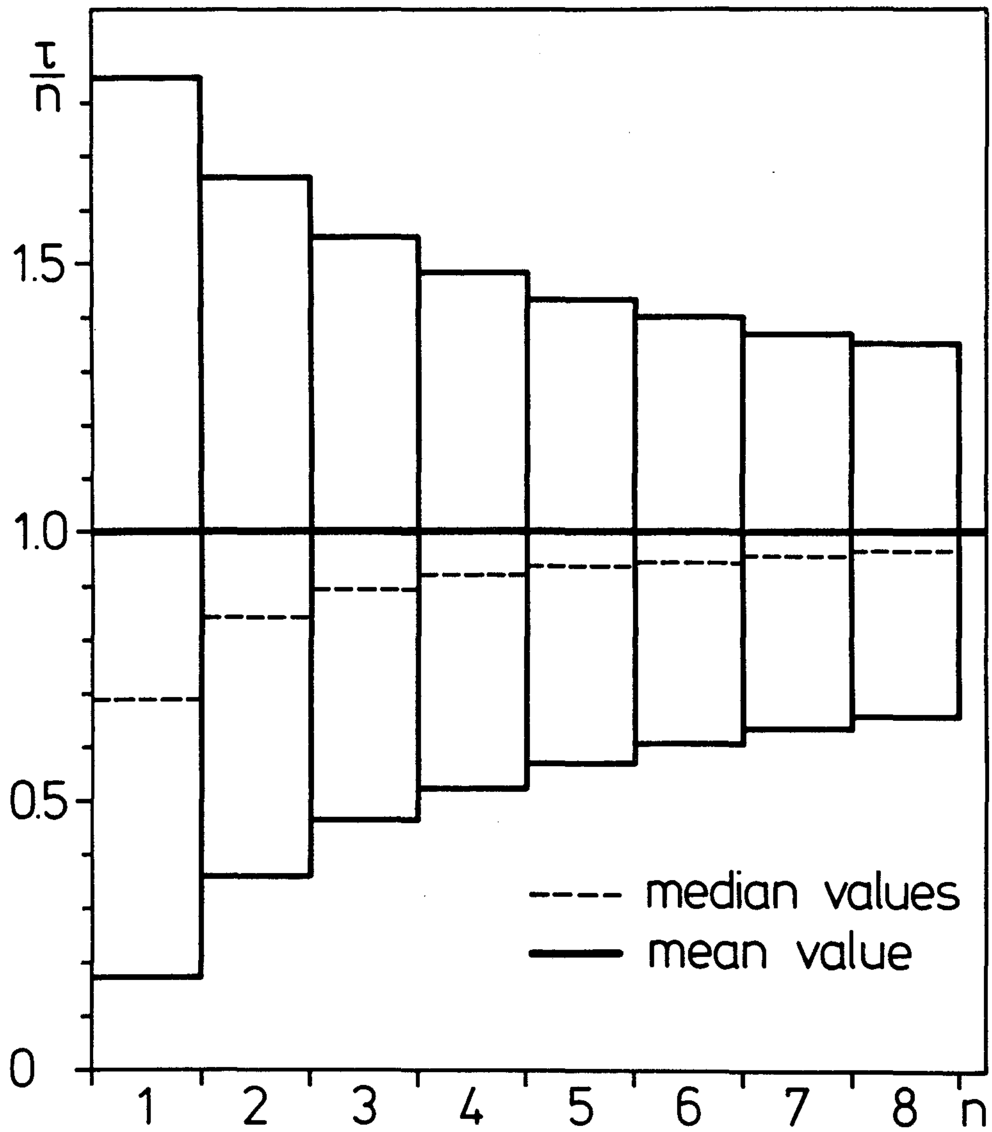

Fig. 5: Median values and 68.3 % confidence intervals for the $g_n(\tau)$ distribution for n = 1 through 8.

of the trend or a trend complement for each interval following a failure. Thus, the time after the last failure is considered; the corresponding STA estimate is $\hat{L}_K$ and from it one obtains the annual rate, $\hat{\Lambda}$:

$$\hat{\Lambda}_K = \hat{L}_K \Omega_K. \tag{45}$$

In a technology, such as nuclear power reactors, were PRA methods are extensively employed, one can assess the system improvements in terms of $\delta\lambda_K^{PRA}$, since the technical specifics of the changes are known. In the case of failure-unrelated improvements, one can directly deduct $\delta\lambda_K^{PRA}$ from the recent Λ estimate. We therefore equate it to a Λ-change:

$$\delta\Lambda_{(K+1)}(t) = \delta\lambda_{(K+1)}^{PRA}(t) \text{ for failure-unrelated changes .} \tag{46}$$

The index (K+1) in parenthesis indicates a location "somewhere" in the K+1st interval, and "t" the time of the change within this interval.

For the failure related changes in the system the Λ-change could also be equated with the PRA-estimate:

$$\delta\Lambda_{(K+1)}^{K}(t) = \delta\lambda_K^{PRA}(t); \tag{47}$$

the superscript K on Λ^K and the subscript K on λ_K indicate the relation to the K-th failure. In this case one can possibly improve the $\delta\Lambda$-estimate by directly using the apparent $\tilde{\lambda}_K$ for the appearance of that particular failure. Suppose this failure occurred for the first time; subsequently more-than-repair action deemed necessary. Then, the entire operational period T , disregarding interruptions of the time scale by other failures, gives the apparent rate

$$\tilde{\lambda}_K = \frac{1}{T_K}, \tag{48}$$

as in Eq. (7). The change of the apparent λ is then obtained by multiplying with the <u>relative</u> change as assessed by PRA, which is the same as multiplying the RHS of Eq. (47) with the ratio of apparent and PRA values of λ:

$$\delta\Lambda^{K}_{(K+1)}(t) = \frac{\tilde{\lambda}_K}{\lambda_K^{PRA}} \delta\lambda_K^{PRA}(t) . \tag{49}$$

It appears that $\delta\tilde{\Lambda}$ per Eq. (49) is preferable to Eq. (47) since $\delta\Lambda^K$ is applied as a change of $\hat{\Lambda}_K$ which in itself contains the K-th failure in its "apparent" timing.

If $\delta\Lambda^*_{(K+1)}(t)$ denotes the Λ-changes through system deployment and decommissioning, and if $\Omega_{(K+1)}(t)$ denotes the corresponding variation of Ω, then one obtains an estimate for the L-complement after the K-th failure prior to the K+1st one (during the K+1 interval):

$$\delta L_{(K+1)}(t) = \frac{\hat{\Lambda}_K + \delta\Lambda_{(K+1)}(t) + \delta\tilde{\Lambda}^K_{(K+1)}(t) + \delta\Lambda^*_{(K+1)}(t)}{\Omega_{(K+1)}(t)} - \hat{L}_K \tag{50}$$

For $t = t_K$, the time of the K-th failure, the δ-quantities disappear. Thus,

$$\delta L_{(K+1)}(t_K) = 0. \tag{51}$$

At the time of the K+1st failure one then obtains - within the uncertainty band of the isotonic regression estimate - an estimate of the average L in the previous interval, which over several intervals should be similar to the trend calculated in Eq. (50). In a single interval, the trend complement per Eq. (50) should be much more accurate than the difference between two isotonic regress: on estimators If e.g. if Δ F_{K+1} is incidentally smaller or equal to the previous ΔT, the value of $\hat{L}_{K+1}$ would be equal to the (newly calculated) $\hat{L}_{K;}$ a calculated trend complement however could indicate a significant improvement for this period.

V. TREND ANALYSIS FOR THE EVOLUTION OF NUCLEAR POWER IN THE U.S.

As discussed in Ref. 2, there were five nuclear reactor failures in the U.S. that provided considerable learning experience for the evolution of commercial nuclear power in the U.S., though three of these failures did not occur on commercial plants. Figure 6 presents the results obtained in Ref. 2 as a heavy solid histogram on a logarithmic scale. The $\hat{L}_k$ values are embedded in a 68.3 % confidence band, calculated for single events from Eq. (34). On the log-scale all of these confidence intervals have the same width, from about 1.84^{-1} to 0.17^{-1} times the mean.

Connecting the lower end of the first interval with the upper end of the last one and vice versa gives lower and upper statistical limits of the trend in the confidence band of a factor of 5 as lower and a factor of 350 as upper statistical trend limits respectively, with a factor of 50 being the mean value. Furthermore, the null hypothesis test (comp. Sec. IV) gives $\alpha = 1$ %, i.e. a 1 % probability for no trend. Thus, H_o is to be rejected. Both findings strongly suggest that the trend is statistically significant.

But, since all five failures led to considerable learning experience, reflected in system improvements, there are sound technical reasons for a DFR-trend, in addition to the statistical evidence.

An analysis of failures in a certain category normally includes IFM and UFM events. Figure 6 shows only UFM failures since there was no contribution of IFM accidents in this class of severity.

The statistical trend analysis may be extended to the period beyond the most recent failure, provided that sufficient operational time has been accumulated. So is, at the time of this writing, the number of operating years of U.S. power reactors since March 1979 (TMI-2 accident) already larger than in the previous interval. This suggests a reduction of the post-TMI-2 failure rate already from a statistical point of view. A more

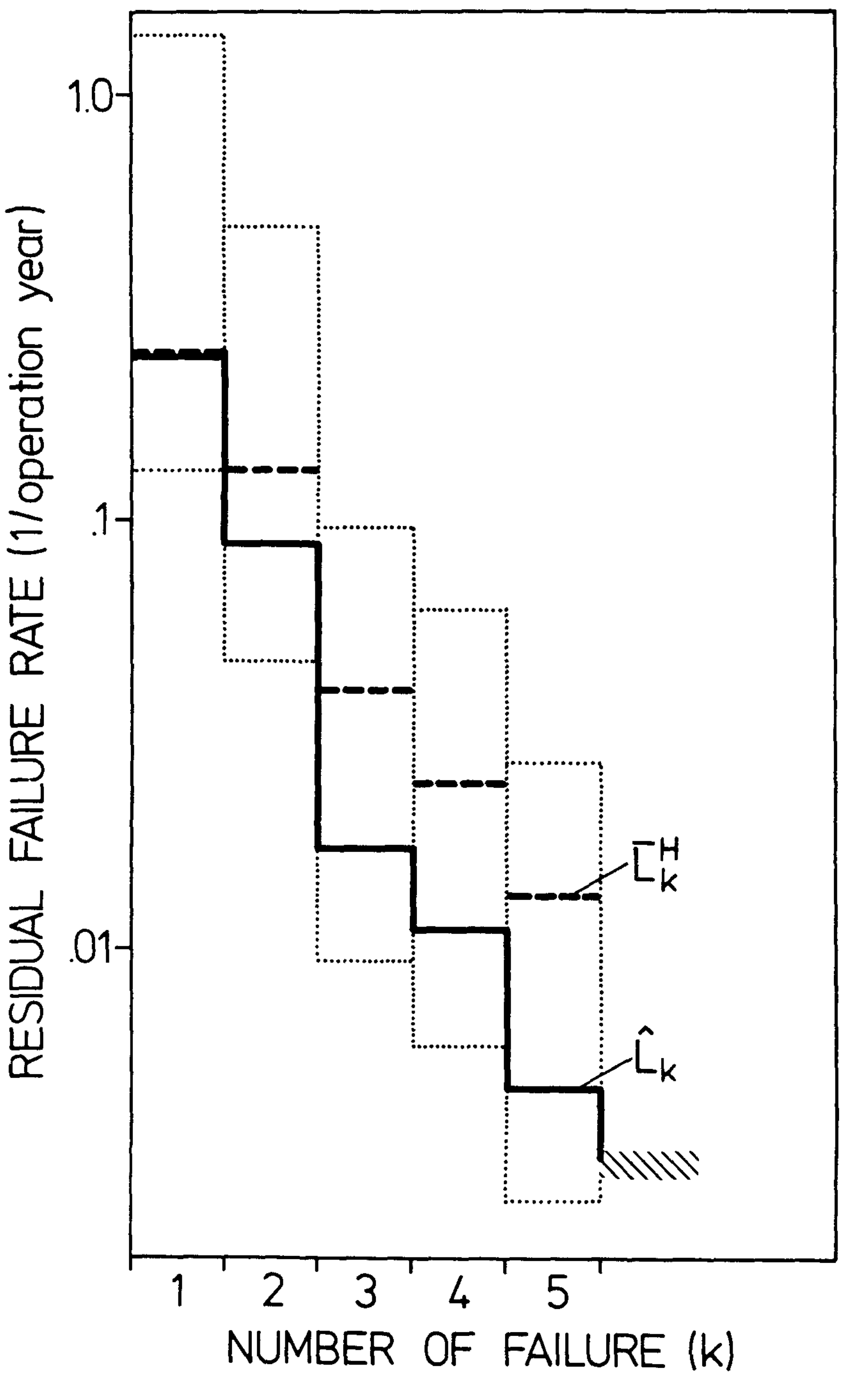

Fig. 6: Residual failure rates of Ref. 7 for category II events on U.S. nuclear reactors (solid lines), completed with 68 %-confidence band (dotted lines), and historical average failure rate $\bar{L}_k^H$ (dashed lines).

meaningful estimate can however be obtained from a PRA-evaluation of the recent system improvements in terms of a complement as described above.

Also plotted in Fig. 6 is a histogram of the historical average values, $\bar{L}_k^H$, that are obtained by dividing all failures by the entire operating period, as given by Eq. (5). It is seen that the $\bar{L}_k^H$-values have also a declining trend though to a considerably lesser degree than the $\hat{L}_k$. Actually, the $\bar{L}_k^H$ drift away from $\hat{L}_k$ on the upside, by about a factor of three at the last failure.

This brings up a more general question, that is also posed for other technologies, such as large dams or boilers aboard steam ships: What is the current expected rate for major failure? Are estimates of that rate as derived from actuarial data better described by the recent values of the average historical rate, $\bar{L}_K^H$, by the STA estimate, $\hat{L}_K$, or by an assessment of the state-of-the-art safety characteristic?

The major difference between the estimates $\bar{L}_K^H$ and $\hat{L}_K$ is in the treatment of UFM failures that happened early in the evolution of a technology: In $\bar{L}_K^H$ all failures have the same contribution if the failure possibility has been retained or if it has been eliminated after the failure. In the trend analysis estimate on the other hand, early failures that occurred with a high apparent rate are noted as such, but they affect later rates only to the extent of their recurrence; if they do not recur, they affect subsequent rates in the sense that the operation times, that determined the early high rates, must not be included in the calculation of later rates. This appears to be a proper way to assess failure rates after system changes.

The third possibility, the assessment of the state-of-the-art safety characteristics does not aim at and thus does not provide a representation of the failure rate of a grown and evolving population of various types of systems. In using it as a projection one disregards earlier failures completely. A sequence of state-of-the-art assessments also provides trend information that is however not directly based on actuarial failure event spacings.

Therefore, the evaluation of the actuarial data in terms of a failure rate trend in the presented manner appears to be a good compromise between the historical average and the state-of-the-art assessments. It accounts fully for the failures in the evolving technology, including early failures with the high rates prevalent at these times, and it allows for system improvements, if demonstrated in the actuarial data, to properly affect the estimates of the most recent failure rates.

SUMMARY AND CONCLUSIONS

As technologies evolve, along with general technical progress as well as through learning from operating experience, failure rates and associated risks vary with time, normally in the direction of improved safety performance. The assessment of this variation in form of a trend of failure rate is of importance for a control of the success of past measures and a basis for decisions on future actions.

The methodology for statistical trend analysis (STA) as presented here is based upon the evaluation of actuarial data on the temporal spacings between failure events, within a certain category. It is a generalization of a somewhat simpler version published earlier. The domain of application is that middle ground between the frequent failures in a population of a large number of identical units and the extremely rare events for which no actuarial information is available. The assessment results in the two latter frequency domains are (well defined) average values of actuarial data on the one hand and PRA-synthesized expected values on the other. In the middle ground one analyzes sparse data that are assembled while a technology, a system or a component continue to change.

The generalizations consist of allowing for partial changes in individual failure rates as well as additions of new failures that manifest themselves in a trend moderation on even a trend reversal. A special analysis procedure for trend reversal is presented.

The event-oriented nature of this consideration is contrasted in a special discussion with the expectation values of PRA. The uncertainties of the PRA-calculated frequencies lead to a modified exponential density function for the occurrence chance of a failure along the time axis. The corresponding confidence intervals are displayed in a frequency vs. consequence (L vs. C) diagram in which the average risk ($\bar{R}$) is shown as a diagonal base line. Individual failure events can be represented by "points," with their apparent consequences and occurrence rates as respective coordinates.

Three categories of actions after failures are considered and represented as domains in this diagram. It is plausible to assume that individual events stimulate "more-than-repair" action or even "design modifications" if they are well above the $\bar{R}$ base-line. This naturally leads to an evolutionary pressure toward system improvement.

The evaluation of the significance of tentatively identified trends is discussed. Statistical evaluations are discussed in the form of an "alternate vs. null hypothesis" test and in form of a confidence band around the trend histogram. In addition, an extensive and detailed evaluation of prior technical information on system changes in terms of a PRA-computed complement to an STA-derived trend is proposed as an independent trend measure. Both quantities should mutually support each other.

A previously published statistical analysis of major failures on U.S. power reactors and developmental precursors was used as an example to illustrate the trend significance information. The trend obtained suggests an average factor of 50 in the reduction of the failure rates with factors of 5 and 350 as extreme boundaries. The null hypothesis evaluation indicates a 1 % probability for the data to be compatible with a constant failure rate. Taking 5 % as a typical criterion indicates that the null hypothesis is to be rejected. This result is supported by the consideration of the system modifications that were implemented either independently or as a result of lessons learned from these failures.

Finally, three trending description concepts were qualitatively compared:

- The historical average failure rate, $\bar{L}^H$, is the least sensitive description, since it retains past failures including the ones that occurred early in the evolution of a technology with an unaltered contribution to the numerator of $\bar{L}^H$.
- The statistical trend analysis result (STA).

- The state-of-the-art projection based on either a single or sequence of several PRA calculations.

The STA result appears to be an adequate compromise that accounts fully for failures during the technology evolution (with high rates if they were prevalent at early times) but it also allows for system improvements to properly affect the estimates of the most recent failure rates, if such improvements are demonstrated in the actuarial data on failure spacings.

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APPENDIX No. 1

The Modified Exponential Density Function

The log-normal distribution

$$p_{ln}\left(\frac{L}{\bar{L}}\right) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{\left(\ln \frac{L}{\bar{L}}\right)^2}{2\sigma^2}\right), \tag{54}$$

normalized as

$$\int_{-\infty}^{\infty} p_{ln}\left(\frac{L}{\bar{L}}\right) d\ln \frac{L}{\bar{L}} = 1 \tag{55}$$

is employed to describe the uncertainty variation of the time constant L in the exponential density function

$$f_o(t) = L\, e^{-Lt}. \tag{56}$$

Averaging Eq. (56) with p_{ln} $(L/\bar{L})$ yields the modified exponential density function as defined in Eq. (13):

$$f_\sigma(t) = \bar{L} \int_{-\infty}^{\infty} \frac{L}{\bar{L}} \exp\left(-\frac{L}{\bar{L}}\,\bar{L}t\right) \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{\left(\ln \frac{L}{\bar{L}}\right)^2}{2\sigma^2}\right) d\ln \frac{L}{\bar{L}}\,. \tag{57}$$

The integral Eq. (57) is calculated numerically as function of $\bar{L}t$ with σ as parameter. The results for three σ-values are presented in Fig. 3. The values for t = 0 can be evaluated analytically:

$$f_\sigma(0) = \bar{L}\, e^{\sigma^2/2}. \tag{58}$$